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Efficient One-way Wave-equation Migration in Tilted Transversally Isotropic Media

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SUMMARY

We present a technique for efficient one-way wave-equation migration of seismic data in Tilted Transversally Isotropic Media. The technique combines explicit and implicit wavefield extrapolation, and achieves good imaging results at a fraction of the computational cost of Reverse Time Migration.

Introduction

Cost-effective and accurate imaging tools for anisotropic media are becoming an integral part of the standard imaging workflow. While methods based on Reverse Time Migration (RTM) have great advantages over one-way methods in terms of handling multipathing, overturned reflections, internal multiples and arbirary dips, one-way methods are still capable of delivering fit-for-purpose solutions at a fraction of the computational cost of RTM. We propose two different algorithms for depth extrapolation in Tilted Transversally Isotropic (TTI) media, using explicit (see e.g. Hale (1991)) and implicit (see e.g. Ristow and Rühl (1997); Nolte (2008)) finite differencing. Both approaches have advantages and can be combined in a highly efficient migration code. We demonstrate that the implicit methods presented in Shan (2007) can be generalised to handle dips close to 90° at a modest increase in computational complexity, and propose a framework for generating and applying very short spatial explicit extrapolators with wavenumber-domain filters that successfully complement implicit methods to achieve a considerable migration speed-up.

TTI Dispersion Relation

We start from the 2-dimensional dispersion relation for plane waves propagating in a Vertically Transversally Isotropic (VTI) medium with coordinates r (horizontal) and w (vertical):

$$\frac{\omega^2}{v_p^2} = (k_r^2 + k_w^2) + \varepsilon k_r^2 - \frac{f \cdot (k_r^2 + k_w^2)}{2} \times \left[1 - \sqrt{\left(1 + \frac{2\varepsilon k_r^2}{f \cdot (k_r^2 + k_w^2)}\right)^2 - \frac{8(\varepsilon - \delta) k_r^2 k_w^2}{f \cdot (k_r^2 + k_w^2)^2}} \right] \qquad (1)$$

This dispersion relation can be obtained from the expression for the velocity of a plane wave in a VTI medium (see Tsvankin (1996); Shan (2007)). In equation (1), v_p is the vertical pressure wave velocity, f is a constant, and ε, δ are the Thomsen parameters. As a TTI medium is just a rotated VTI medium, we introduce two additional angle parameters: the azimuth ψ and tilt ϕ of the TI symmetry axis. Expressing the wavenumbers k_r and k_w via k_x, k_y, k_z and using (1), we can solve the vertical wavenumber k_z as an implicit function of k_x and k_y and the medium parameters, $k_z = K(\omega/v_p, \varepsilon, \delta, \phi, \psi, k_x, k_y)$. The function K is multivalued, but we will always use only one branch (hence the name *one-way equation*) that we compute and tabulate numerically. Depth extrapolation of a wavefield $u(\omega, x, y, z)$ in a heterogenous TTI medium consists in solving the following pseudo-differential operator equation:

$$\partial u/\partial z = iK\left(\omega/v_p(\overset{2}{x},\overset{2}{y},z), \varepsilon(\overset{2}{x},\overset{2}{y},z), \delta(\overset{2}{x},\overset{2}{y},z), \phi(\overset{2}{x},\overset{2}{y},z), \psi(\overset{2}{x},\overset{2}{y},z), i\overset{1}{\partial/\partial x}, i\overset{1}{\partial/\partial y}\right) u \qquad (2)$$

where medium parameters are arbitrary functions of the subsurface coordinates x, y, z, and indices over the operators indicate order.

Explicit Extrapolator

In explicit extrapolation, we approximate the explicit *depth-marching* scheme $u(x,y,z+\Delta z) = e^{i\Delta z K} u(x,y,z)$ with a finite-difference operator (*explicit extrapolator*) for every set of medium parameters $\omega/v_p, \varepsilon, \delta, \phi, \psi$

$$u(x,y,z+\Delta z) = \sum_{|m|,|l| \le L} a_{ml}\left(\omega/v_p, \varepsilon, \delta, \phi, \psi\right) e^{m\Delta x \partial/\partial x l \Delta y \partial/\partial y} u(x,y,z) = P\left(\partial/\partial x, \partial/\partial y\right) u(x,y,z) \qquad (3)$$

where L is the *aperture* of the explicit extrapolator (3). Given a maximum imaged dip angle α, we call the set of points $\Pi_\alpha = \{(k_x, k_y) : (k_x^2 + k_y^2)/k_z^2 <= \tan^2\alpha\}$ the *passband*, and its complement – the *cutband* Γ. We require the finite-difference operator in the right-hand side of (3) to have an absolute value of strictly less than one in the wavenumber doman over the cutband, thus forcing the cutband modes to decay with depth. The explicit extrapolator (3) can be constructed in a variety of ways, however, the main challenge is to achieve accuracy over the passband and stability for all wavemodes while minimising the aperture L. This can be achieved through the solution of a constrained convex optimisation problem $\|\exp\{i\Delta z K\} - P(ik_x, ik_y)\| \to \min \quad k_x, k_y \in \Pi_\alpha, \; |P(ik_x, ik_y)| < H(k_x, k_y) \quad k_x, k_y \in \Gamma$

where H is a barrier function defined over the cutband, controlling the cutband modes' rate of decay. Solving this problem with the constraint over the entire cutband could be burdensome and result in larger operator apertures. We propose to impose the constraint only on a smaller circular region $C(\tau) = \{(k_x,k_y) : \sqrt{k_x^2+k_y^2} <= (1+\tau)\,\mathrm{diam}\,(\Pi_\alpha)/2\}$ with $\tau = \tau(\omega/v_p,\varepsilon,\delta,\phi,\psi) > 0$ (see Fig. 1 for an example for $\alpha = 60°$, $\omega = 30$Hz, $L = 3$, $\tau = .2$, $\varepsilon = .45$, $\delta = .2$, $v_p = 1000$, $\phi = 60°$, $\psi = 40°$). The effect of this is that explicit extrapolators built for different values of medium parameters may still have substantially overlapping stability domains. The resulting spatial finite-difference operators (3) can then be applied in groups of operators that are stable over the largest passband, followed by a single cutband filter application per group. The computational cost of this algorithm per depth step (excluding the one-off computation of the finite-difference coefficients of (3)) in the absence of sharp velocity contrasts can be roughly estimated as

$$\left[\max_{(x,y)} v_p(x,y,z)/\left((1+\tau)\min_{(x,y)} v_p(x,y,z)\right)+1\right] \times \mathrm{FFT}(N_x,N_y) + (2L+1)^2 N_x N_y \tag{4}$$

where N_x, N_y are inline and crossline grid sizes, and FFT denotes the cost of a two-dimensional FFT. In case of sharp velocity contrasts (e.g., in the presence of salt bodies) the first term in equation (4) should be replaced with a sum of similar terms corresponding to each velocity range of relatively smooth lateral variation near a reference velocity. Our tests indicate that a pre-computed coefficient look-up table containing single-precision variable-aperture explicit 3D extrapolator coefficients, of a size not larger than the memory required for a 3D velocity model, is adequate for accurate interpolation of extrapolator coefficients at run-time. It should be noted that for a fixed maximum dip α the aperture L grows with the ratio ω/v_p. However, an appropriate choice of τ and the barrier function H allows us to achieve reasonably compact operators even for large values of the ratio – see Fig. 1.

Implicit Extrapolator

We approximate the right-hand side of equation (2) straight with a finite-difference operator, rather than a rational function of wavenumbers. For example, in 2D we have:

$$K(\omega/v_p,\varepsilon,\delta,\phi,k_x) \approx \sum_{m=-N_1}^{m=N_1} a_m(\omega/v_p,\varepsilon,\delta,\phi)e^{im\Delta x k_x} \Big/ \sum_{m=-N_2}^{m=N_2} b_m(\omega/v_p,\varepsilon,\delta,\phi)e^{im\Delta x k_x} \tag{5}$$

The coefficients a_m and b_m in equation (5) can be computed using nonlinear least squares over the set Π_α for a specified maximum dip angle α. If $M = \max(N_1,N_2)$ is the problem *bandwidth* then subsequent solving of (2),(5) using trapezoidal rule is equivalent (in the two-dimensional case) to solving a system of linear equations with a $2M+1$-diagonal matrix. The *LU* factorisation of such a matrix can be performed in roughly $2N_xM^2$ operations (see e.g. Demmel (1997)), and subsequent solution by backsubstitution requires roughly $6N_xM$ operations. For 3D, assuming $\Delta x = \Delta y$, we can approximate equation (2) using multiway splitting (see Ristow and Rühl (1997)). Note that increasing the bandwidth typically enhances the accuracy of approximation (5) by an order of magnitude and increases the maximum imaged dip angles – see Fig. 2 where dispersion and error curves are plotted for 2D,$M = 1,\ldots,5$, $\omega = 30$Hz,$\varepsilon = .45$,$\delta = .2$,$v_p = 1000$,$\phi = 60°$. Although the complexity of solving a system of linear equations with a $2M+1$-diagonal matrix grows quadratically with M, the quadratic part is contributed by the factorisation algorithm that can be performed once for each model, but the numerical complexity of backsubstitution that is performed at run-time grows only linearly. To get rid of parasitic modes arising due to arbitrary behaviour of the approximation (5) outside of the passband, at each depth step we filter out all energy beyond the maximal passband of operators (2). Tests that we have run so far only required a single application of a wavenumber-domain filter, even for models with sharp contrasts. The numerical complexity of the resulting algorithm per depth step, on the assumption that only one filter application is required, can be be roughly estimated as

$$2\mathrm{FFT}(N_x,N_y) + 32MN_xN_y \tag{6}$$

in the notations of equation (4).

Implementation and Examples

We have implemented the above algorithms in the framework of 2D and 3D *shot-record migration* in highly parallelised Fortran 2003 codes. The implicit finite-differencing turns out to be more efficient at the same dip performance for larger values of ω/v_p, while the explicit extrapolation requires very short operators ($L = 1,2$) for small temporal frequencies and large velocities. However, the size of implicit finite differences grows slower with maximum dip than the aperture of the explicit extrapolator, making it the method of choice where dip performance close to 90° is required (see Fig. 2). The two methods can be successfully combined in one code, with implicit finite differences used to propagate waves through top layers of smaller seismic velocities, and the explicit extrapolator taking over whenever the ratio ω/v_p makes it a more economical choice. Fig. 3 and 4 demonstrate the results of applying a hybrid 60° maximum dip implicit-explicit migration code to the 2007 BP TTI Synthetic Model. Fig. 3 shows fragments of the density model for a salt body and an anticlinal structure. Fig. 4 shows the results of hybrid migration, demonstrating a perfect agreement between reflector locations on the density model and the image. Note, however, that the dip limitation results in poor performance of the one-way extrapolation close to salt flanks. Increasing the maximum dip angle doesn't always help, as accurate imaging of some events requires imaging of overturning waves (see Albertin et al (2002)). However, in terms of performance, the algorithms presented here offer an order-of-magnitude speed-up compared to the existing implementations of RTM. One-way methods can be easily combined with coordinate rotation to achieve partial imaging of dips beyond 90° in one direction (see Shan et al. (2007)). Moreover, our method can be combined with a curvilinear coordinate transformation (e.g., along a ray path) for beam-wave imaging (see Albertin et al (2002); Brandsberg-Dahl and Etgen (2003)).

Conclusions

We have described computationally efficient and stable depth extrapolation methods based on a one-way equation for TTI media. Explicit and implicit methods can be combined in a hybrid technique to speed up depth migration and reduce splitting errors. One-way extrapolation in TTI media can be used in beam-wave imaging methods to achive partial imaging of steep and overturning events. All that, in combination with an order-of-magnitude lower cost than that of RTM, makes the proposed methods ideal for target-oriented imaging and where high-speed data turnaround is of higher importance than the multipathing capabilities of RTM.

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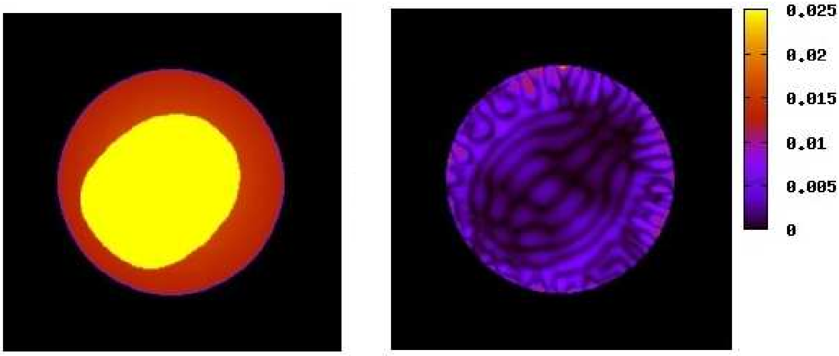

Figure 1 *3D explicit extrapolator* $\Pi_\alpha, C(\tau)$ *(left) and approximation error (right). The colour scale applies to the right plot only.*

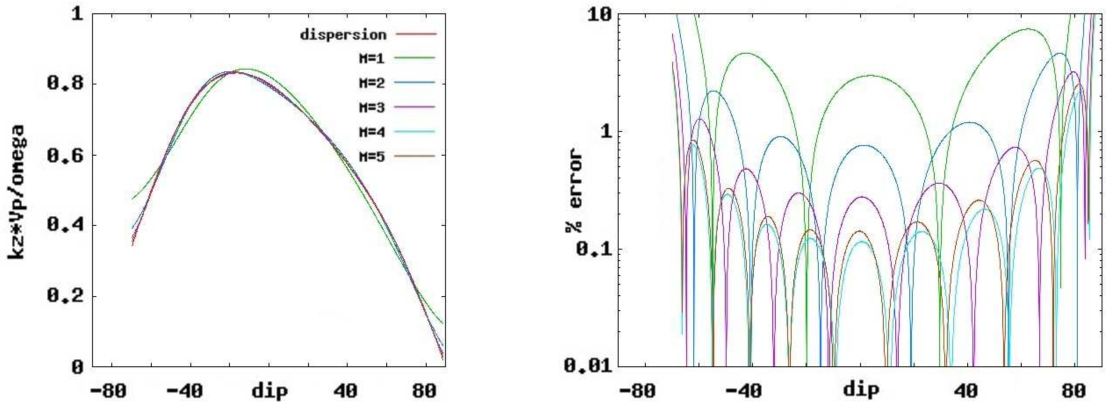

Figure 2 *2D implicit extrapolator curves (left) and approximation error (right, logarithmic scale) for various matrix bandwidth values.*

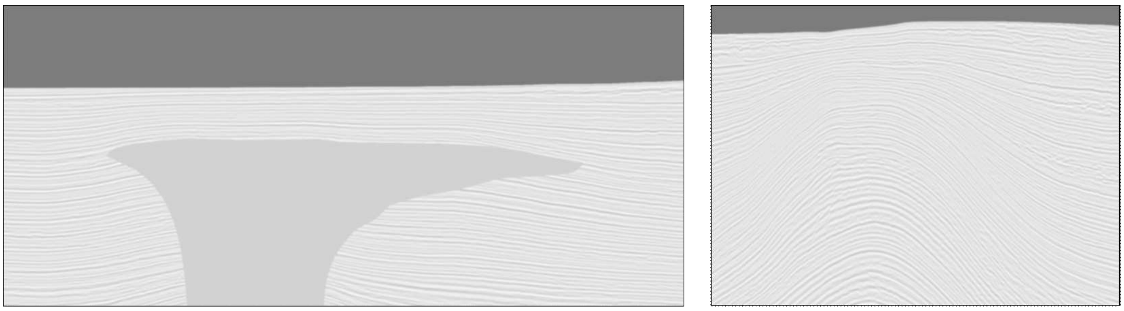

Figure 3 *BP TTI model: density* ρ *for salt and anticlinal regions.*

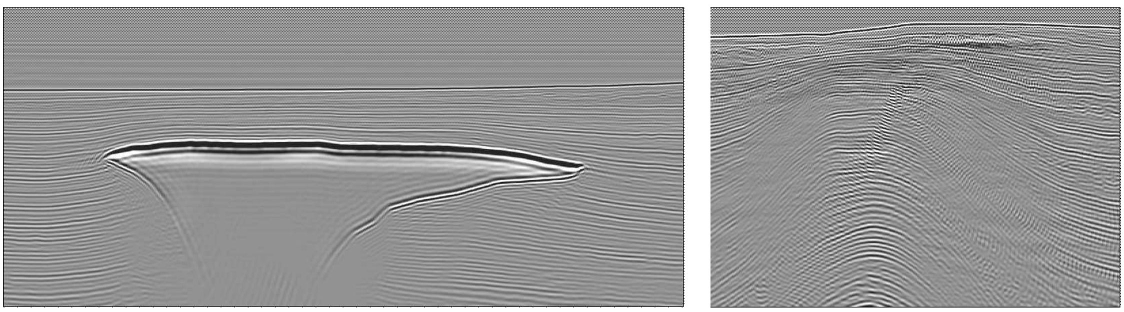

Figure 4 *BP TTI model:* 60° *maximum dip, one-way shot-record migration image.*