Joint 4DFWI with model difference regularization

Musa Maharramov (maharram@stanford.edu) and Biondo Biondi (biondo@sep.stanford.edu), Stanford University

Motivation: improved reservoir monitoring

- Depends on successful tracking of fluid movement
- Conversion of time shifts $\Rightarrow$ impedance changes
- Requires manual interpretation
- More automated approaches based on, e.g., WE image-difference tomography (Albertin et al., 2006; Maharramov and Albertin, 2007)
- 4D FWI is a potentially powerful 4D tool (Routh et al., 2012) but sensitive to repeatability issues (Asnaashari et al., 2012; Zheng et al., 2011; Rakness et al., 2013)
- Goal: design robust 4DFWI, less sensitive to repeatability issues

Existing 4D FWI methods

- *Parallel Difference* FWI (iterated) (Miller et al., 2007)

$$\mathbf{m}_{\mathrm{INIT}} \begin{cases} \rightarrow \text{BASE FWI}^n \Rightarrow \mathbf{m}_b \\ \rightarrow \text{MON FWI}^n \Rightarrow \mathbf{m}_m \end{cases} \rightarrow \Delta\mathbf{m} = \mathbf{m}_m - \mathbf{m}_b$$

- *Sequential Difference* FWI (iterated) (Oldenborger et al., 2007)

$$\mathbf{m}_{\mathrm{INIT}} \rightarrow \text{BASE FWI}^n \Rightarrow \mathbf{m}_b \rightarrow \text{MON FWI}^n \Rightarrow \mathbf{m}_m \rightarrow \Delta\mathbf{m}$$

- *Double Difference* (Watanabe et al., 2004)

$$\| (\mathbf{M}_m^s \mathbf{u}_m - \mathbf{M}_b^s \mathbf{u}_b) - (\mathbf{M}_m \mathbf{D}_m - \mathbf{M}_b \mathbf{D}_b) \| \rightarrow \min, \quad (1)$$

where $\mathbf{M}_{m,b}^{(s)}$ are equalization or measurement operators, and $\mathbf{u}_{m,b}$, $\mathbf{D}_{m,b}$ are synthetic and actual data.

Joint 4DFWI: simultaneous inversion and cross-updating

- NEW: *Simultaneous FWI* of baseline and monitor with *difference regularization* (Maharramov and Biondi, 2013; Ayeni and Biondi, 2012):

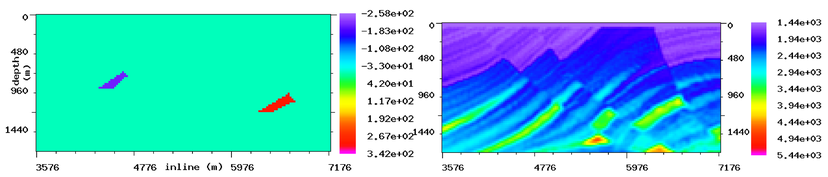
$$\sum_{i=1}^{2} \alpha_i \|\mathbf{W}_d^i [\mathbf{D}_i - \mathbf{u}(\mathbf{m}_i)]\|^2 + \quad (2)$$

$$+ \alpha \|\mathbf{R}\mathbf{W}(\mathbf{m}_2 - \mathbf{m}_1 - \Delta\mathbf{m}^{\mathrm{P}})\|^2. \quad (3)$$

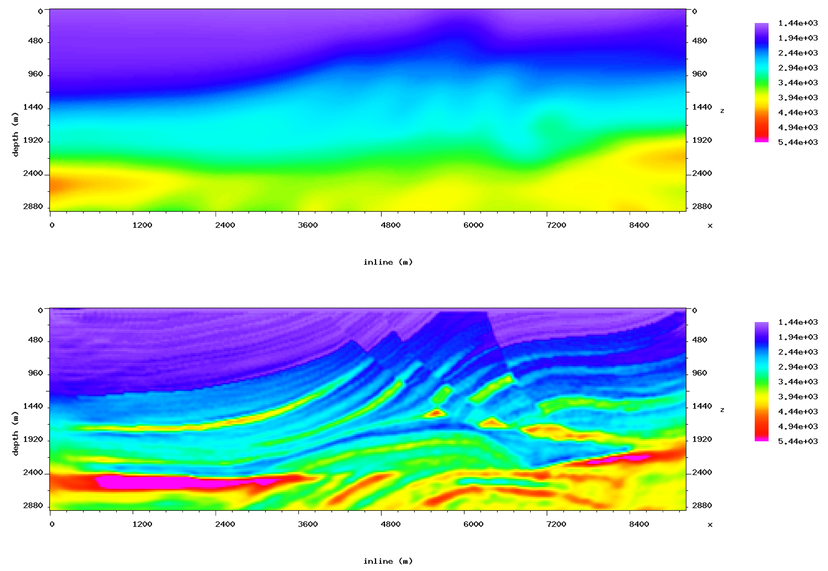
- NEW: *Cross-updating* Approximation to the Simultaneous FWI (Maharramov and Biondi, 2013):

$$\mathbf{m}_{\mathrm{INIT}} \rightarrow \text{BASE FWI} \Rightarrow \mathbf{m}_b \rightarrow \text{MON FWI} \Rightarrow \mathbf{m}_m$$
$$\mathbf{m}_m \Leftarrow \text{MON FWI} \quad \mathbf{m}_b \Leftarrow \text{BASE FWI}$$
$$\rightarrow \Delta\mathbf{m} \leftarrow$$

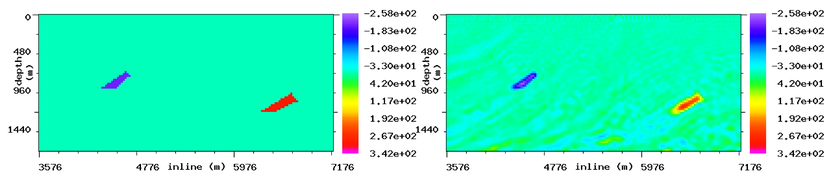
True difference and inverted target area



Starting baseline vs 3-15.5 Hz full baseline FWI result



True difference vs cross-updating with non-repeatable surveys



Why it works?

Assuming that the error in the baseline model estimate $\widehat{\mathbf{m}}_b$ contains a component $\delta\mathbf{m}_m$ and the error in the monitor model estimate $\widehat{\mathbf{m}}_m$ contains a component $\delta\mathbf{m}_b$, the Parallel Difference yields

$$\widehat{\Delta\mathbf{m}} = \widehat{\mathbf{m}}_m - \widehat{\mathbf{m}}_b \approx \Delta\mathbf{m} + \delta\mathbf{m}_b - \delta\mathbf{m}_m,$$
$$\delta\mathbf{m}_b \in \mathrm{null}\left(\frac{dF}{d\mathbf{m}}(\mathbf{m}_b)\right), \ \delta\mathbf{m}_m \in \mathrm{null}\left(\frac{dF}{d\mathbf{m}}(\mathbf{m}_m)\right). \quad (4)$$

The Sequential Difference propagates $\delta\mathbf{m}_m$ into $\widehat{\mathbf{m}}_m$, resulting in

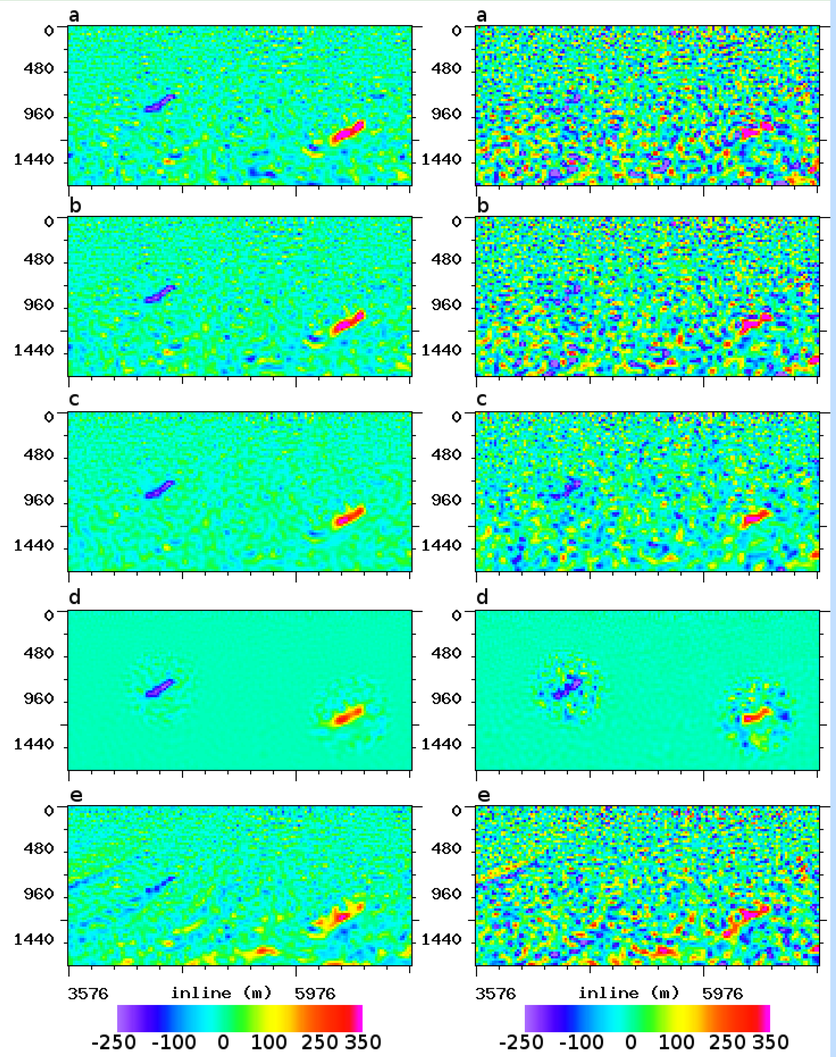
$$\widehat{\Delta\mathbf{m}} = \widehat{\mathbf{m}}_m - \widehat{\mathbf{m}}_b \approx \Delta\mathbf{m} + \delta\mathbf{m}_b + \delta\mathbf{m}_m - \delta\mathbf{m}_m = \Delta\mathbf{m} + \delta\mathbf{m}_b. \quad (5)$$

The cross-updating and simultaneous inversion propagate the error components $\delta\mathbf{m}_m$ and $\delta\mathbf{m}_b$ into *both* models, canceling them in the difference:

$$\widehat{\Delta\mathbf{m}} = \widehat{\mathbf{m}}_m - \widehat{\mathbf{m}}_b \approx \Delta\mathbf{m} + \delta\mathbf{m}_b + \delta\mathbf{m}_m - \delta\mathbf{m}_m - \delta\mathbf{m}_b = \Delta\mathbf{m}, \quad (6)$$

where $\Delta\mathbf{m}$, $\widehat{\Delta\mathbf{m}}$ are the *recoverable* and estimated model differences.

4DFWI of noisy data (14 and 7 dB SNR)



(a) parallel diffference; (b) sequential difference;
(c) cross-updating; (d) simultaneous inversion; (e) double-difference.

Summary and perspectives

- Both cross-updating and simultaneous 4DFWI perform well on noisy synthetic data.
- **W** in (3) can be constructed from oscillations of alternating cross-updates (Yang et al., 2014)
- Next: test the simultaneous 4DFWI (2,3) with **W** from *geomechanical and rock physics* constraints.