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## Full Waveform Inversion for Reservoir Monitoring - Pushing the Limits of Subsurface Resolution

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### SUMMARY

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Fluid movement in the subsurface and the associated changes in saturation translate into changes of the subsurface elastic parameters. Stress changes, whether due to fluid extraction/injection or deformation such as slips on preexisting faults, affect the elastic parameters as well. Detecting and inverting the imprint of changing subsurface elastic parameters on seismic data lies at the heart of time-lapse seismic imaging for reservoir monitoring. In this work we demonstrate that the recently proposed technique of simultaneous time-lapse full-waveform inversion with a model-difference regularization can be used to extract high-resolution information on magnitude and location of subsurface velocity and stress anomalies, potentially providing valuable input for reservoir monitoring and assessment of geohazards.

## Introduction

A recently developed methodology for time-lapse full-waveform inversion (FWI) based on a simultaneous inversion with a total-variation (TV) model-difference regularization (Maharramov et al., 2015, 2016) has been demonstrated to achieve multi-scale inversion of subsurface velocity changes in the presence of strong repeatability issues. The main objective of this work is to demonstrate applicability of this technique to subsurface stress and geohazard monitoring. More specifically, using a synthetic example with added noise, we demonstrate that reliable indicators of the subsurface stress change and locations of stress anomalies can be extracted from seismic data. The following empirical relation is often used to describe the dependence of the subsurface velocity on the effective stress in reservoir rocks (Johnston, 2013),

$$V = V_{\infty} \left( 1 - A \exp - \frac{P}{P_0} \right), \quad (1)$$

where  $V$  is acoustic velocity,  $P$  is isotropic effective stress, and  $V_{\infty}$ ,  $A$ , and  $P_0$  are positive fitting constants for various types of rocks (Domenico, 1977; Zimmer, 2003; Lee, 2003; Johnston, 2013). Equation (1) means that velocities increase in “compacting” rocks with increasing effective stress, flattening out at a high effective stress—see, for example, Figure 20 from Chapter 3 of Johnston (2013). Typically, a few-megapascal change in the effective stress results in a few tens of meters-per-second change in the acoustic velocity within the affected rocks. Can such relatively small changes of the acoustic velocity be detected from seismic data? We show that the simultaneous FWI with a TV model-difference regularization can indeed reliably resolve such small changes in the presence of noise.

## Theory

In our method, we invert the baseline and monitor models simultaneously by solving the following optimization problem:

$$\text{minimize } \alpha \| \exp i \arg \mathbf{u}_b - \exp i \arg \mathbf{d}_b \|_2^2 + \beta \| \exp i \arg \mathbf{u}_m - \exp i \arg \mathbf{d}_m \|_2^2 + \quad (2)$$

$$\delta \| \mathbf{R}(\mathbf{m}_m - \mathbf{m}_b) \|_1 \quad (3)$$

with respect to both the baseline and monitor models  $\mathbf{m}_b$  and  $\mathbf{m}_m$ . Problem (2,3) describes a time-lapse FWI with a model-difference regularization (3) (Maharramov et al., 2015). When  $\mathbf{R} = \nabla$ , the regularization term (3) represents a total-variation (TV) regularization that promotes “blockiness” of the model difference, potentially reducing oscillatory artifacts (Rudin et al., 1992). If  $\mathbf{R} = \mathbf{I}$  is the identity map, we obtain a sparsity-promoting  $L_1$  model-difference regularization. Problems with  $\mathbf{R} = \nabla$  and  $\mathbf{R} = \mathbf{I}$  can be solved in a cascaded fashion, resolving “blocky” changes first, followed by spiky velocity-difference anomalies. The data misfit terms (2) are formulated in the frequency domain and represent misfits between the amplitude-normalized observed and predicted wave fields, in other words representing phase misfits. Maharramov et al. (2016) showed that minimization of the phase misfits in (2) is to a first order equivalent to a tomographic inversion of the travel-time delays due to changes in the subsurface model. Regularization (3) plays a dual role in our method: it penalizes oscillatory artifacts in the model difference that may be due to acquisition and computational repeatability issues, and it constrains the inverted model by fitting the sparsest model difference that explains the data. Since problem (2,3) is to a first order equivalent to a tomographic model-difference inversion, conceptually we can study the limits of its resolution by considering the constrained tomographic inversion problem

$$\begin{aligned} \| \mathbf{A} \delta \mathbf{s} - \delta \tau \|_2 &< \sigma, \\ \| \delta \mathbf{s} \|_0 &= k, \end{aligned} \quad (4)$$

where  $\delta \tau$  is a vector of observed time shifts,  $\delta \mathbf{s}$  is the unknown slowness change,  $\mathbf{A}$  is the travel-time modeling operator,  $\sigma$  is the 2-norm of the estimated measurement error, and  $\| \cdot \|_0$  is the  $L_0$  norm (the number of non-zero components) of a vector. In problem (4) we fit the observed time shifts with a slowness difference of a given sparsity (see Elad (2010) for a discussion of the relation between  $L_0$  and  $L_1$ -regularized optimization). If  $\delta \mathbf{s}_0$  is a true solution of (4) for some observed time shifts  $\delta \tau$ , any minimizer  $\delta \mathbf{s}$  of (4) satisfies the estimate:

$$\| \delta \mathbf{s} - \delta \mathbf{s}_0 \|_2 \leq \frac{2}{c_{2k}} \sigma, \quad (5)$$

where  $c_{2k}$  is the lower restricted isometry constant of operator  $\mathbf{A}$  (Candes et al., 2006) defined as

$$c_{2k} = \min_{J:|J|=2k} \lambda_{\min}(\mathbf{A}_J), \quad (6)$$

where  $J$  is a subset of  $2k$  columns of  $\mathbf{A}$ ,  $\mathbf{A}_J$  is the operator made up of those columns,  $\lambda_{\min}$  is its minimal singular value. Equation (5) relates the recoverability of a  $k$ -sparse slowness model to acquisition parameters and noise in the data. Note that simply increasing dimension of the data (the number of receivers) does not improve estimate (5): the singular value in (6) and the  $L_2$  norm of the noise grow at the same asymptotic rate with increasing number of receivers so long as noise distribution of an individual time-shift measurement is the same. However, according to the Central Limit Theorem, redundant measurements reduce the noise level: for the temporal average of  $N$  repeated measurements at the same receiver locations

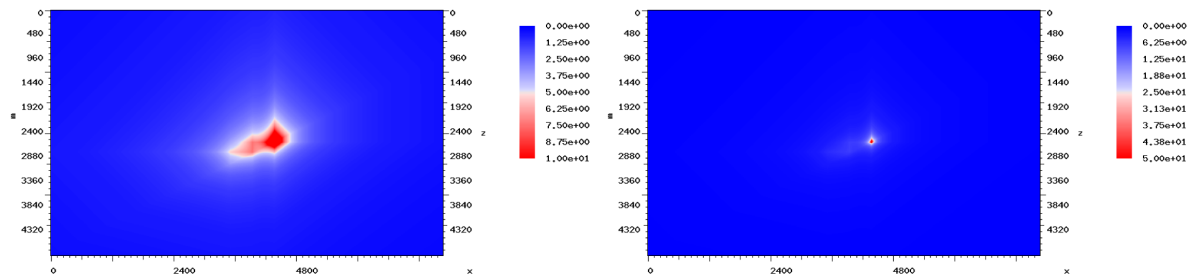
$$\delta\tau = \frac{1}{N} \sum_{i=1}^N \delta\tau^i, \quad (7)$$

the corresponding measurement error in the first line of (4) decreases asymptotically as  $\sigma/\sqrt[4]{N} \rightarrow 0$ . In other words, given sufficiently redundant observations, we should be able to recover the geometry and (qualitatively) the magnitude of a sparse model difference. Averaging in (7) does not necessarily mean multiple surveys; we envisage the use of emergent continuous-source technologies (Kurosawa and Kato, 2015) as a cost-effective alternative to repeated surveys. It should be noted that the magnitude of the subsurface slowness change is recovered qualitatively because the regularization results in a penalization of the model difference; however, enhancements exist that can address this phenomenon (Maharramov et al., 2016). One potentially important implication of estimate (5) for velocity-stress relations of the form (1) is that continuous observations in combination with a robust simultaneous FWI (2,3) can detect relative magnitudes of the subsurface stress changes, such as a “flattening” of (1) for large changes in the effective stress. Our method is merely an inversion tool for detecting small changes in the subsurface, but enhanced monitoring capabilities that can be delivered by this technology open up new, if somewhat speculative at this point, possibilities. For example, can a “flattening” of the velocity-stress curve near a locked fault undergoing stress change due to natural or man-made phenomena indicate an impending slip? While study of viable precursors is well beyond the scope of this work, the inversion technique that we propose may prove instrumental in both conventional and novel approaches to subsurface monitoring.

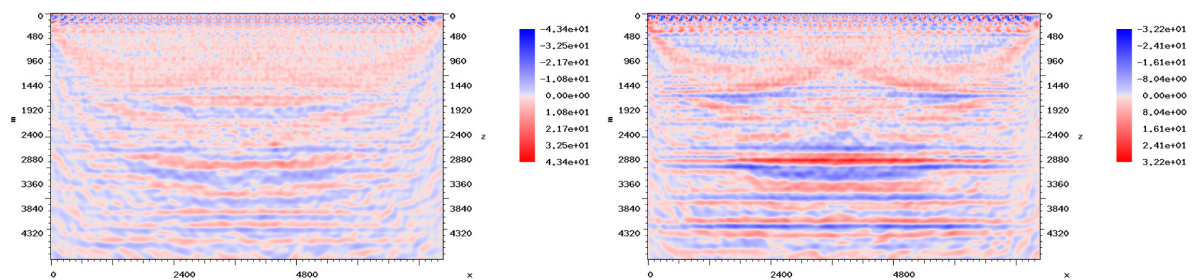
## Examples

We demonstrate the high-resolution power that can be achieved by our method (2,3) on the synthetic baseline model used by Maharramov et al. (2016). We assume very small velocity changes between the first monitor and baseline surveys (Figure 1), and velocity changes between the second and first monitor surveys that have half the magnitude of the changes between the first monitor and the baseline. The figures show the true velocity change between the first monitor and baseline surveys at a 10 m/s and 50 m/s clip that is modeled to imitate the effect of the stress change near a locked segment of a fault undergoing an interseismic slip, with a singularity near the tip of the locked segment. Note that both compressive and tensile stresses are typically present around a locked fault segment, and according to (1) that means both positive and negative velocity changes (Segall, 2010). However, in this experiment we use only a single positive lobe of the true velocity change. Naïve application of FWI often results in non-physical oscillatory artifacts, or “side lobes”, that in our case can be easily mistaken for the effects of stress regime changing from compressive to tensile. Therefore, we demonstrate the robustness of our method (2,3) by recovering the right sign of a positive model difference without producing oscillatory artifacts. The clean synthetic was generated using 39 shots at a 192 m spacing and 320 receivers per shot with a 24 m receiver spacing. Absorbing boundary conditions were applied at the top of the model to avoid surface-related multiples, and a Ricker wavelet centered at 12 Hz was used as the source. FWI was conducted with a frequency continuation from 4 to 20 Hz, starting from a smoothed model of Maharramov et al. (2016). First, we conducted a parallel-difference FWI after adding random noise to the data. Signal-to-noise ratio (SNR) of the noisy data peaked at about 8 dB; however, the SNR deteriorated to 1 at 4 and 20 Hz. The results of the parallel-difference FWI are shown in Figure 2. Note that the second model difference is completely masked by oscillatory artifacts while the difference on the left panel only hints at the location of the stress singularity and is contaminated with oscillatory artifacts

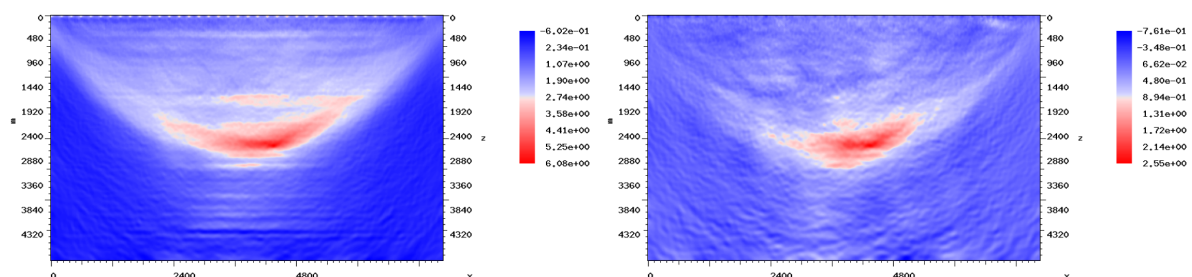
of significant magnitude that render it uninterpretable. Next we applied the simultaneous FWI with a TV model-difference regularization. Solving (2,3) with  $\mathbf{R} = \nabla$  largely recovered the velocity changes and their relative magnitudes (see Figure 3), although the absolute magnitudes were underestimated. The TV regularization flattened the velocity peak, resulting in a more ambiguous location of the stress singularity. However, supplying the result of the TV-regularized simultaneous inversion as the starting model for problem (2,3) with a sparsity-promoting  $L_1$  regularization ( $\mathbf{R} = \mathbf{I}$ ) resulted in the recovery of the sparse velocity peak corresponding to the stress singularity—see Figure 4.



**Figure 1** True model difference between the first monitor and base at 10 (left) and 50 (right) m/s clip.



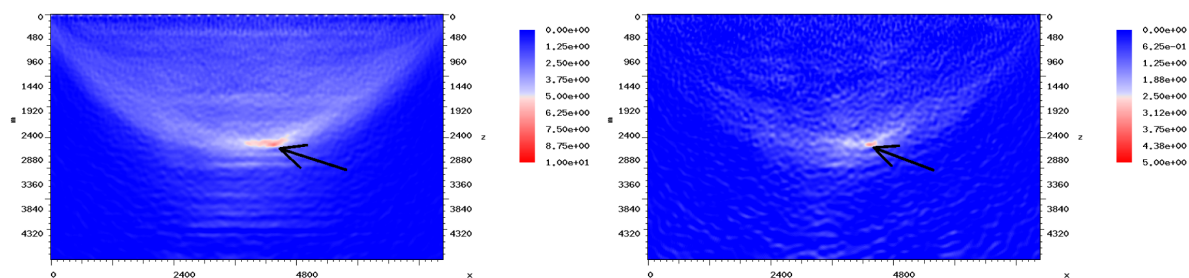
**Figure 2** Parallel-difference inversion of the first (left) and second (right) difference from noisy synthetics.



**Figure 3** First (left) and second (right) difference inversion from noisy data using simultaneous FWI with a TV model-difference regularization.

## Conclusions

Given continuous or repeating seismic observations, simultaneous time-lapse FWI with a sparsity-promoting model-difference regularization can be used to detect small changes in the subsurface model induced by changes in the stress field. Even in the presence of strong noise, model-difference regularization removes oscillatory artifacts from the inverted model difference while retaining useful information.



**Figure 4** First (left) and second (right) difference inversion from noisy data using cascaded inversion with a TV and  $L_1$  model-difference regularization.

Cascaded inversion with TV and  $L_1$  regularization helps to achieve multi-scale inversion of subsurface changes, potentially pinpointing locations of significant stress change. While absolute values of the inverted model difference are underestimated due to regularization, relative magnitudes are indicative of changing rates of stress change, and may be of value in seismic hazard studies.

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