AN ANALOGUE OF KOLMOGOROV-PETROVSKY-PISKOUNOV THEOREM FOR QUASILINEAR PARABOLIC EQUATIONS

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We study Cauchy problem for the following quasilinear parabolic equation:

$$u_t = (K(u)u_x)_x + F(u) \tag{1}$$

where $K(0) = 0$, $K(u) > 0, u > 0$; $K(u) \in C^\infty((0,\infty)) \cap C([0,\infty))$; $K(u) = Cu^\sigma + o(u^\sigma)$, $u \to 0, \sigma > 0, C > 0$; $F(0) = F(1) = 0$, $F(u) \in C^\infty([0,+\infty))$, $F'(1) < 0$ with monotonely increasing initial data $u(0,x) = u_0(x)$, $0 \leqslant u_0(x) \leqslant 1, x \in \mathbb{R}$ that satisfy a uniform Holder condition, and where the solution is such that the flux is continuous. In this work we prove a generalisation of KPP for equation (1). The Cauchy problem is reduced to a mixed problem for the flux $W(u(x,t),t) = K(u(x,t))\,u_x(x,t)$ as a function of u and t: $W_t = K^{-1}(u)W^2(u)\left[W_u + R(u)W^{-1}\right]_u$ with initial and boundary-value data $W(0,t) = W(1,t) = 0$; $W(u,0) = W_0(u), 0 \leqslant u \leqslant 1$ wherein $u_t = W_u(u,t)u_x + F(u)$, $W_0(u_0(x)) = K(u_0(x))\,u_{0x}(x)$ and $R(u) = F(u)K(u)$. Equation (1) has a monotone travelling wave solution $u(x,t) = \theta(x + \lambda t)$ that corresponds to the stationary flux as a function of u. Spectral study of the differential operator $-\frac{d^2}{du^2} + \frac{1}{2}\frac{d}{du}\left(R(u)\tilde{W}^{-2}\right)$ where $\tilde{W}(u)$ is the stationary flux, led (see [1]) to the following

Theorem 1. *Under the above conditions assume that $u_0(x)$ is sufficiently close to $\theta(x)$ in $L_2(\mathbb{R})$. Solution to the Cauchy problem converges then "by shape and speed"to the travelling wave - e.g., $(\forall\tau \in (0,1))$: $K(u(\phi^\tau(t),t))\,u_x(\phi^\tau(t),t) \to \tilde{W}(\tau)$, $t \to +\infty$ where $\tilde{W}(u)$ is the stationary flux and $\phi^\tau(t)$ is such that $u(\phi^\tau(t),t) = \tau$. Furthermore, $\phi^\tau(t)(t\lambda)^{-1} \to 1$, $t \to +\infty$ where λ is speed of the travelling wave.*

1. Maharramov M., An Analogue of KPP for Quasilinear Parabolic Equations. *Transactions of the National Academy of Sciences of Azerbaijan. Physical and Mathematical Series.*(2001)**21**, No. 2.,77-80.